

Constraining Leptogenesis from Laboratory Experiments

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ABSTRACT

The presently available information on neutrino oscillations can be used to tightly constrain the light neutrino mass matrix m_ν under the assumption that the three known neutrinos provide explanation of the solar and atmospheric anomalies. We exploit left right symmetry and the seesaw model to establish direct correlation between this constrained m_ν and the mass matrix M_R for the right handed (RH) neutrinos. Using this correlation, one could directly relate the lepton asymmetry ϵ generated in the decay of the lightest RH neutrino to experimentally determined mixing angles of the light neutrinos. It is found that the parameters required to understand neutrino anomalies also give rise to the ϵ in the range required to reproduce the correct baryon asymmetry in the leptogenesis scenario. Specifically, the small angle MSW or the vacuum oscillation explanation of the solar anomaly can lead to correct ϵ for maximal value of the CP violating phases. In contrast, the large angle MSW solution needs some suppression in CP violating phases in order to generate the right ϵ .

It is a well-known hypothesis [1] that the observed asymmetry in the Baryon number (B) of the universe owes its existence to lepton number (L) or lepton flavour violating [2] interactions occurring at temperatures much greater than the weak scale. According to this hypothesis-known as Leptogenesis-the asymmetry in Lepton number is created by the out of equilibrium L and CP violating interactions at a high scale [1]. This can get converted to the Baryon asymmetry by the presence of non-perturbative $B + L$ violating interactions [3] which are believed to be in thermodynamic equilibrium between the temperature $T \sim 10^2 - 10^{12}$ GeV . These interactions respect $B - L$ and wash out any asymmetry in $B + L$ leaving a residual B asymmetry Y_B [4]

$$Y_B = CY_{B-L} , \quad (1)$$

C being a constant of order one determined by the content of the theory at the electroweak scale.

The above scenario for Leptogenesis can be nicely realized in the presence of heavy right handed (RH) Majorana neutrinos [1,5]. The existence of these neutrinos at a high scale is indirectly hinted by the observed (mass)² difference of the light left handed neutrinos at the Superkamioka [6]. The scale $m_{\nu_\tau} \sim 10^{-1} - 10^{-2}$ eV finds a natural explanation in terms of the seesaw model with a right handed neutrino having mass near the grand unification scale. If the right handed neutrinos also display hierarchy similar to the left handed ones then the decay of the lightest RH neutrino may be responsible for generation of the lepton asymmetry [1,5]. The value of the Y_{B-L} generated this way however depends upon the mixing and masses among the right handed neutrinos. These are a priori arbitrary. In contrast, the masses and mixing among the light left handed neutrinos get strongly constrained if one demands that the three light neutrinos simultaneously explain the observed solar and the atmospheric neutrino deficits. If one could use the underlying left right symmetry to relate the constrained spectrum of the left handed neutrinos to that of the right handed ones, then the lepton asymmetry could be related to the low energy parameters. We wish to discuss here a possible scenario where such relation comes out naturally.

Let us consider conventional left right symmetric picture with three right handed neutrinos N_i , ($i = 1, 2, 3$) mixing among each other. The Dirac Yukawa couplings of N to the leptonic doublets l leads to the decay $N \rightarrow \phi l$ and $N \rightarrow \phi^\dagger l^c$. The corresponding rates are asymmetric in the presence of CP violation. The lepton asymmetry is generated in this picture by the out of equilibrium decay of the lightest right handed neutrino. The decay of this neutrino leads to a CP asymmetry ϵ [5]

$$\epsilon = \frac{3}{16\pi v^2} \frac{1}{(m_D^\dagger m_D)_{11}} \sum_{j=2,3} \text{Im} \left[(m_D^\dagger m_D)_{1j}^2 \right] \frac{M_1}{M_j} . \quad (2)$$

m_D denotes the Dirac type neutrino mass term and M_i are the masses of the RH neutrinos. $v \sim 174$ GeV denotes the weak scale. We assumed hierarchical masses M_i in writing the above ϵ . The ϵ gets resonantly enhanced if two of the RH neutrinos are nearly degenerate [8]. We shall not consider this case and concentrate on the above ϵ .

The lepton asymmetry generated in the decay is related approximately to ϵ [5] as follows:

$$Y_L \sim \kappa \frac{\epsilon}{g^*} \sim (10^{-3} - 10^{-4}) \epsilon , \quad (3)$$

where $g^* \sim 100$ are the effective degrees of freedom present in the cosmic plasma at $T \sim M_1$. The κ is a dilution factor arising due to other lepton number violating processes which tend to erase the lepton asymmetry in the decay. Typical value of κ following from the numerical solution of the relevant Boltzmann equations is around $10^{-1} - 10^{-2}$ [5] leading to the value quoted on the RHS of eq. (3). In view of the uncertainty in some of the low energy parameters we shall not attempt to solve the exact Boltzmann equation and determine κ . Instead, we shall take κ to be in the range $10^{-1} - 10^{-2}$ obtained in other analyses [5]. This then requires through eq.(3) that one needs $\epsilon \sim (10^{-6} - 10^{-7})$ in order to generate $Y_B \sim 10^{-10}$. Our main aim is to relate ϵ to masses and mixing of the light neutrinos and see if one could get an ϵ in this range.

The CP asymmetry ϵ in the decay of a neutrino can lead to appreciable Y_L if it is out of equilibrium at the time of its decay. This requires that

$$K \equiv \frac{\Gamma_1}{H} = \frac{(m_D^\dagger m_D)_{11} M_1}{8\pi v^2 H} \leq 1 , \quad (4)$$

H being the Hubble parameter. The above equation places an important constraint on the scale of the left right symmetry breaking as we shall see.

Without assuming any specific model, we shall use the general structure for the neutrino masses that would follow in seesaw model with a left right symmetry. The light neutrino masses in such scheme can be described by [9]

$$m_\nu \sim m_{LL} - m_D M_R^{-1} m_D^T . \quad (5)$$

Here m_{LL} (M_R) are Majorana masses for the left (right) handed neutrinos and m_D are Dirac masses connecting them. The assumption of the left right symmetry implies

$$m_{LL} \equiv v_L f \quad M_R \equiv v_R f \quad , \quad (6)$$

where $v_R(v_L)$ are the (triplet) vacuum expectation values (vev) which set the scale of the right (left) handed neutrino masses and f is a Yukawa coupling matrix. One therefore has

$$M_R = \frac{v_R}{v_L} m_{LL} \quad . \quad (7)$$

The vevs $v_{R,L}$ are not independent in a large class of models but are related by [10]

$$v_L v_R \sim \gamma M_W^2 \quad , \quad (8)$$

γ being a model-dependent parameter. Eq.(8) implies that the v_L and hence the first term m_{LL} in eq.(5) displays seesaw structure similar to the second term. m_{LL} is normally presumed to be small and is neglected in conventional seesaw model. As emphasized in [11], m_{LL} can play an important role in simultaneous solution of the solar and atmospheric neutrino problems through seesaw mechanism. Eqs. (5,6,8) are our crucial assumptions which would allow us to fix the structure of M_R . We shall also assume that m_D is determined by the up quark masses and assume that the mixing among the up quarks and charged leptons is small enough to be neglected.

We shall not presuppose any specific structure for the Yukawa coupling matrix f but determine it by requiring that the resulting m_ν explains the solar and atmospheric anomalies simultaneously. The relevant scales can be identified as follows. The dominant contribution of the seesaw term is of $O(\frac{m_L^2}{v_R})$. Requiring that this is not larger than the atmospheric scale $\sim 10^{-1}$ eV gives $v_R \geq 10^{14}$ GeV, close to the grand unification scale. Eq.(8) then implies that v_L is also close to the atmospheric neutrino scale. Thus the contribution of the first term can be comparable to the largest contribution from the conventional seesaw term and it is more appropriate to retain both these terms. We shall do that in the following.

Before proceeding further, let us collect relevant information on elements of m_ν following from experiments [7]. The present experimental results emerging from the data on atmospheric neutrinos at Superkamioka and various solar neutrino detectors can be reconciled in three neutrino framework either by assuming a hierarchical or almost degenerate neutrino mass spectrum. We shall concentrate in this paper on the hierarchical spectrum. The present experimental results are strong enough to constrain elements of the mixing matrix U_L . The U_L relates the flavour $\alpha = e, \mu, \tau$ and mass $i = 1, 2, 3$ eigenstates as follows:

$$\nu_\alpha = U_{L\alpha i} \nu_i \quad . \quad (9)$$

After appropriate redefinition of the charged lepton states, the matrix U_L contains three mixing angles and three physical phases and can be parameterized by [12]

$$U_L = W_{23}(\theta_{L23}) W_{13}(\theta_{L13}) W_{12}(\theta_{L12} e^{i\delta}) P(\lambda_i) \quad , \quad (10)$$

where $P(\lambda_i) = \text{diag.}(e^{i\lambda_1}, e^{i\lambda_2}, 1)$; W_{ij} denote complex rotation in the ij plane. For example,

$$W_{12}(\theta_{L12} e^{i\delta}) \equiv \begin{pmatrix} c_{L12} & s_{L12} e^{-i\delta} & 0 \\ -s_{L12} e^{i\delta} & c_{L12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad . \quad (11)$$

The δ in the above equation is analogous to the Kobayashi Maskawa phase while $\lambda_{1,2}$ arise due to Majorana nature of neutrinos. The latter cannot be probed through the conventional neutrino oscillation experiments but would play an important role in generating the lepton asymmetry in the decay of the right handed neutrinos as we shall see.

The constraints on different elements in this case have been discussed in [7] and we choose parameters according to values given in the first of [7]. The simplifying feature of this analyses is strong restriction implied by the CHOOZ experiment on the angle s_{L13} [7]:

$$|U_{Le3}|^2 \sim s_{L13}^2 \leq 5 \times 10^{-2} \quad . \quad (12)$$

This restriction is valid for $\Delta m_{31}^2 \geq 3 \cdot 10^{-3} \text{ eV}^2$.

Relative smallness of s_{L13} together with hierarchy in masses imply that the survival probabilities for the solar and atmospheric (ATM) neutrino data assume two generation like form and restrict the mixing angles s_{L12}, s_{L23}

respectively. The restrictions however depend upon the chosen solution for the solar neutrino anomaly and there are three possibilities namely, small angle MSW [14] (SAMSU), large angle MSW (LAMSU) and the vacuum oscillations (VAC). The relevant restrictions as presented for example in [7] are as follows:

$$\begin{aligned}
|U_{Le2}| &\approx s_{L12} \approx 0.03 - 0.05, & (\text{SAMSU}) \\
&\approx 0.35 - 0.49, & (\text{LAMSU}) \\
&\approx 0.48 - 0.71. & (\text{VAC}) \\
|U_{L\mu3}| &\approx s_{L23} \approx 0.49 - 0.71. & (\text{ATM})
\end{aligned} \tag{13}$$

The two neutrino masses $m_{2,3}$ under the assumption of the hierarchy in them get constrained as follows:

$$\begin{aligned}
m_2^2 &\approx \Delta m_{SUN}^2 \approx (0.4 - 1.2) \cdot 10^{-5} \text{ eV}^2, & (\text{SAMSU}) \\
&\approx (0.8 - 3.0) \cdot 10^{-5} \text{ eV}^2, & (\text{LAMSU}) \\
&\approx (0.6 - 1.1) \cdot 10^{-10} \text{ eV}^2. & (\text{VAC}) \\
m_3^2 &\approx \Delta m_{ATM}^2 \approx (0.4 - 6.0) \cdot 10^{-3} \text{ eV}^2. & (\text{ATM})
\end{aligned} \tag{14}$$

It is seen that apart from the phases, the mixing matrix U_L is strongly restricted by the current experiments. This allows reliable determination of the possible neutrino mass matrix and then of M_R through the left right symmetry. In what follows we shall allow s_{L13} to vary between 0.0-0.22. However for analytic study we shall often neglect s_{L13} and concentrate on the following approximate form of U_L :

$$U_L \sim \begin{pmatrix} c_{L12}e^{i\lambda_1} & s_{L12}e^{i(\lambda_2-\delta)} & 0 \\ -s_{L12}c_{L23}e^{i(\delta+\lambda_1)} & c_{L12}c_{L23}e^{i\lambda_2} & s_{L23} \\ s_{L12}s_{L23}e^{i(\delta+\lambda_1)} & -c_{L12}s_{L23}e^{i\lambda_2} & c_{L23} \end{pmatrix}. \tag{15}$$

Given the masses and mixing as in eq.(13,14), we could determine m_ν as

$$m_\nu = U_L^* \text{diag.}(m_1, m_2, m_3) U_L^\dagger. \tag{16}$$

This in turn can be used to determine the Yukawa coupling matrix f and hence M_R . Determination of M_R in terms of the constrained low energy parameters then directly leads to ϵ .

It is seen from eqs. (5,6,8) that ordinary seesaw contribution is suppressed compared to m_{LL} only if the coefficient γ is chosen much larger than 1. In the most natural situation with $\gamma \sim O(1)$ both m_{LL} and the top quark contribution to the second term of eq.(5) are comparable. We thus retain the seesaw contribution but we shall include only the dominant top quark contribution. This contributes only to $(m_\nu)_{33}$ in eq.(5) when mixing among the up quarks is neglected. Thus the Yukawa coupling matrix f may be written as:

$$\begin{aligned}
f_{ij} &= \frac{(m_\nu)_{ij}}{v_L}, & \text{when both } i \text{ and } j \text{ are } \neq 3 \\
f_{33} &= \frac{(m_\nu)_{33} + s}{v_L}.
\end{aligned} \tag{17}$$

The parameter s refers to the contribution arising from the seesaw term. This can be self consistently determined by inserting eq.(17) in eq.(5). One finds

$$s \sim \frac{v_L m_t}{\sqrt{\gamma} M_W} + O\left(\frac{v_L m_1}{m_3 s_{L12}^2}\right). \tag{18}$$

As expected s is comparable to $(m_\nu)_{33}$ for $\gamma \sim O(1)$ and $v_L \sim m_3$. We have checked numerically that approximation made in eq.(17) is self-consistent, i.e. with f determined from eqs.(17), seesaw contribution to elements of m_ν other than the 33 is sub dominant for $v_R \sim 10^{16}$ GeV and $\gamma \sim O(1)$.

Eqs.(6,17,18) together determine M_R in terms of the low energy parameters. This then allows us to obtain masses and mixing among the right handed neutrinos. Let U_R be defined as:

$$U_R^T M_R U_R = \text{diag.}(M_1, M_2, M_3). \tag{19}$$

Parameterize U_R as

$$U_R = W_{23}(\theta_{R23}e^{i\phi_{R23}}) \ W_{13}(\theta_{R13}e^{i\phi_{R13}}) \ W_{12}(\theta_{R12}e^{i\phi_{R12}}) \ D_R. \quad (20)$$

M_R determined from eq.(5), eq.(6) and eq.(17) can be diagonalized by above U_R and its parameters can be determined in terms of elements of U_L and s :

$$\begin{aligned} \tan 2\theta_{R23} &\approx \frac{m_3}{s} ; \tan \phi_{R23} \approx \frac{sm_2c_{L12}^2 \sin 2\lambda_2}{m_3(m_3 + s)} , \\ \tan 2\theta_{R13} &\approx -\sin 2\theta_{L12} \frac{\sqrt{2}m_2c_{R23}Z_-}{(m_3Z_+ + 2s)} ; \phi_{R13} \approx \delta - 2\lambda_2 , \\ \tan 2\theta_{R12} &\approx \sin 2\theta_{L12} \frac{\sqrt{2}m_2c_{R23}Z_+}{m_3Z_-} ; \phi_{R12} \approx \delta - 2\lambda_2 . \end{aligned} \quad (21)$$

$$D_R = \text{diag.}(e^{-i(\delta-\lambda_2)}, 1, 1) . \quad (22)$$

The masses of the RH neutrino are determined as:

$$\begin{aligned} M_1 &\approx \frac{v_R}{v_L} m_2 s_{L12}^2 , \\ M_2 &\approx \frac{v_R}{v_L} \frac{m_3 Z_-}{2} , \\ M_3 &\approx \frac{v_R}{v_L} \left(\frac{1}{2} m_3 Z_+ + s \right) . \end{aligned} \quad (23)$$

In the above equations,

$$Z_{\pm} \equiv 1 \pm \frac{\sqrt{m_3^2 + s^2} - s}{m_3} .$$

In deriving the above, we have assumed $m_1 \ll m_2$ and specialized to the U_L , eq.(15) with maximal s_{L23} . Non-leading terms of $O(\frac{m_2}{m_3}, \frac{m_2}{s})$ are neglected above. As is evident, the mixing pattern among RH neutrinos is completely determined in terms of parameters of U_L and the seesaw contribution $s \sim m_3$. In the absence of the seesaw contribution s , one would have got $U_R = U_L$ and $M_i = \frac{v_R}{v_L} m_i$. This is changed considerably by inclusion of s . The mixing angle θ_{R23} is large but not maximal. θ_{R12} is suppressed compared to the corresponding θ_{L12} . A small θ_{R13} is induced although corresponding θ_{L13} was chosen zero. The mass of the lightest neutrino is governed in this approximation by the mass m_2 of the second generation neutrino rather than m_1 which is neglected in the above derivation. The RH neutrino masses also display strong hierarchy in general. However it is seen from eqs.(23) that M_2 could become comparable to M_1 for some range of parameters in case of the large angle MSW solution to the solar neutrino problem.

The out of equilibrium constraint on N_1 decay, eq.(4) now implies

$$M_1 \geq (2.8 \cdot 10^{16} \text{ GeV}) |U_{R31}|^2. \quad (24)$$

Using M_1 and U_{R31} from eq.(21,23), we get,

$$v_R \geq 2 (2.8 \cdot 10^{16} \text{ GeV}) \frac{v_L m_2}{m_3^2} c_{R23}^2 \left(\frac{s_{23} Z_+}{Z_-} - \frac{c_{R23} Z_-}{Z_+ + 2s/m_3} \right)^2 . \quad (25)$$

We have retained only the leading contribution due to top quark in writing eq.(24) and eq.(27) below. Since $v_L \sim m_3$ and factor in the last bracket is $O(1)$ for $s \sim m_3$ one obtains

$$v_R \geq (2.8 \cdot 10^{16} \text{ GeV}) \frac{m_2}{m_3} . \quad (26)$$

The out of equilibrium condition is seen to be satisfied easily with $v_R \leq M_{GUT}$ independent of the chosen solution for the solar neutrino anomaly.

The ϵ of eq.(2) is now given by

$$\begin{aligned} \epsilon &\sim \frac{3}{16\pi} \left(\frac{m_t}{v} \right)^2 \sin 2\lambda_2 \left[c_{R12}^2 s_{R23}^2 \frac{M_1}{M_2} + c_{R23}^2 \frac{M_1}{M_3} \right] , \\ &\sim \frac{3}{8\pi} \left(\frac{m_t}{v} \right)^2 \sin 2\lambda_2 \left[\frac{m_2 s_{L12}^2}{m_3} \right] \left[\frac{c_{R12}^2 s_{R23}^2}{Z_-} + \frac{c_{R23}^2}{Z_+ + 2s/m_3} \right] , \\ &\sim 10^{-1} \sin 2\lambda_2 \left[\frac{m_2 s_{L12}^2}{m_3} \right] O(1) . \end{aligned} \quad (27)$$

This asymmetry crucially depends upon the phase λ_2 arising due to the Majorana nature of neutrino. This phase cannot be constrained from the oscillation data. Apart from this, ϵ is determined by the known low energy parameters. The factor in the last bracket in eq.(27) is suppressed both for the MSW as well as the vacuum solution to the solar neutrino problem. Typical value of the ϵ in these cases is given by

$$\begin{aligned}\epsilon &\sim 5 \times 10^{-6} \sin 2\lambda_2, & (\text{VAC}) \\ &\sim 10^{-6} \sin 2\lambda_2, & (\text{SAMS}) \\ &\sim 5 \times 10^{-4} \sin 2\lambda_2. & (\text{LAMS})\end{aligned}\tag{28}$$

It is seen that except for the case of the large angle MSW solution one gets the required ϵ for maximal value of the CP violating phase.

We have neglected contribution of the s_{L13} in the above analytic discussion. This angle is in fact allowed to be larger than the s_{L12} in case of the small angle MSW solution. Moreover, we have assumed $m_3 \sim s$ in the above estimates. We now study variation of ϵ numerically without making these approximations on s and s_{L13} .

The input values for s_{L23} and m_3 are fixed by the atmospheric neutrino anomaly. The values for s_{L12}, m_2 depend upon the chosen solution for the solar neutrino problem. We consider all three possibility namely small angle MSW, large angle MSW and vacuum solutions. The variations of $|\epsilon|$ (eq.(2)) and K (eq.(4)) with s_{L13} and s are displayed in Figs. 1-3 respectively for three different possibilities mentioned above. These figures correspond to assuming large phases namely $\delta = \pi/6, \lambda_1 = \pi/3$ and $\lambda_2 = \pi/4$. The ϵ is largely insensitive to chosen δ, λ_1 .

Remarkably, for the experimentally determined parameters, one is able to get required ϵ and also satisfy out of equilibrium constraint namely $K \leq 1$. ϵ obtained in case of the SAMS (Fig. 1) and VAC (Fig. 3) is in the correct range when CP violating phases are chosen large as in Figs.(1-3). In contrast, the LAMS (Fig. 2) gives larger ϵ and thus would require non-maximal phases if appropriate Y_B is to be generated. This feature is understood from the approximate expression, eq.(27) which shows that ϵ is suppressed in case of VAC by smaller mass ratio. The latter is larger for the MSW case but this is compensated by a smaller s_{L12} in case of SAMS. Similar compensation is not obtained for the LAMS and one thus gets relatively larger ϵ .

It is seen that K and ϵ increase with decrease in the ordinary seesaw contribution measured by s . The normal seesaw contribution s plays an important role in keeping neutrinos out of equilibrium and in generating appreciable ϵ . Numerical studies [5] have shown that one does not strictly need $K \leq 1$ and appreciable Y_L can be generated even when $K \sim O(10)$. This happens in the figure typically for $s \geq 0.1m_3$. We have plotted these figures assuming $v_R = 10^{16}$ GeV. Smaller value for v_R will lead to larger K than the one displayed in the figures.

There have been number of earlier papers [13] which have tried to obtain the structure of the RH masses from theoretical assumptions. These works concentrated on definite textures for the quark and lepton masses and tried to arrive at an M_R which could explain neutrino anomalies. Our main emphasize here was the important contribution due to left handed triplet Higgs which arise in any generic seesaw model based on the left right symmetric theory. This symmetry has in fact played a very important role in our analyses. Unlike the earlier works, we have been able to obtain expression (see, eq.(27)) for ϵ directly in terms of the light neutrino masses and mixing and a model dependent seesaw contribution $s \sim m_3$.

Let us now summarize the basic approach and results of the analyses presented in this paper. The anomalies observed in atmospheric and the solar neutrinos find their natural explanation in terms of the standard left right symmetric theory with three light and three massive RH neutrinos. The experimental observations constrain the structure of the neutrino mass matrix fairly strongly under theoretical assumptions of very small mixing among charged leptons. We have shown here that it is possible to relate the structure of the left handed neutrino mass matrix so determined to that of the right handed neutrino mass matrix in a class of models with left right symmetry. This allows us to correlate the lepton asymmetry generated in the decay of the lightest RH neutrino to experimentally inferred mixings among the light neutrino states. It is indeed gratifying that this correlation is successful, i.e. the set of parameters needed to explain neutrino anomalies can also easily account for the lepton asymmetry of the required magnitude without fine tuning in the relevant CP violating phase.

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- [1] M. Fukugita and T. Yanagida, *Phys. Rev.* **D42** (1990) 1285.
 - [2] E. Kh. Akhmedov, V. A. Rubakov and A. Yu. Smirnov, hep-ph/9803255.
 - [3] V.A. Kuzmin, V.A. Rubakov and M.E. Shaposhnikov, *Phys. Lett.* **B155** (1985) 36.
 - [4] E. W. Kolb and M. Turner, *The Early Universe*, (Addison Wesley, Redwood City, CA, 1990).
 - [5] M. A. Luty, *Phys. Rev.* **D45** (1992) 455. Recent review and original references can be found in A. Pilaftsis, hep-ph/9812256; W. Buchmuller, hep-ph/9812447; U. Sarkar, hep-ph/9906335.
 - [6] Y. Fukuda et al., hep-ex/9805006; hep-ex/9805021 and hep-ph/9807003; T. Kajita, in *Neutrino 98*, Proc. of the XVIIIth Int. Conf. on Neutrino Physics and Astrophysics, Takayama, Japan (June 1998).
 - [7] S. M. Bilenky and C. Giunti, *Phys. Lett.* **B444** (1998) 379; M. Narayan, G. Rajasekaran and S. Uma Sankar, *Phys. Rev.* **D58** (1998) 031301.
 - [8] M. Flanz, E.A. Paschos and U. Sarkar, *Phys. Lett.* **B345** (1995) 248; M. Flanz, E.A. Paschos and U. Sarkar and J. Weiss, *Phys. Lett.* **B389** (1996) 693; M. Flanz, E.A. Paschos, *Phys. Rev.* **D58** (1998) 113009; L. Covi and E. Roulet, *Phys. Lett.* **B399** (1997) 113; A. Pilaftsis, *Phys. Rev.* **D56** (1997) 5431.
 - [9] See e.g. "Massive Neutrinos in Physics and Astrophysics", R. N. Mohapatra and Palash B. Pal, World Scientific (1991).
 - [10] R. N. Mohapatra and G. Senjanovic, *Phys. Rev.* **D23** (1981) 165;
 - [11] D. Caldwell and R. N. Mohapatra, *Phys. Rev.* **D48** (1993) 3259; A.S. Joshipura, *Zeit. fur Physik* **C64** (1994) 31; B. Brahmachari and R. N. Mohapatra, *Phys. Rev.* **D58** (1998) 15003;
 - [12] C. Giunti, C.W. Kim and M. Monteno, hep-ph/9709439.
 - [13] W. Buchmuller and M. Plumacher, *Phys. Lett.* **B389** (1996) 73; M. Plumacher, hep-ph/9704231; W. Buchmuller and M. Plumacher, hep-ph/9904310; M. Plumacher, *Zeit. fur Physik* **C74** (1997) 549; W. Buchmuller and T. Yanagida, hep-ph/9810308; M. Berger and B. Brahmachari, hep-ph/9903406; John Ellis, S. Lola and D. V. Nanopoulos, hep-ph/9902364.
 - [14] L. Wolfenstein, *Phys. Rev.* **D17** (1978) 2369; S. Mikheyev and A. Yu. Smirnov, *Sov. J. Nucl. Physics*, **42** (1985) 913.

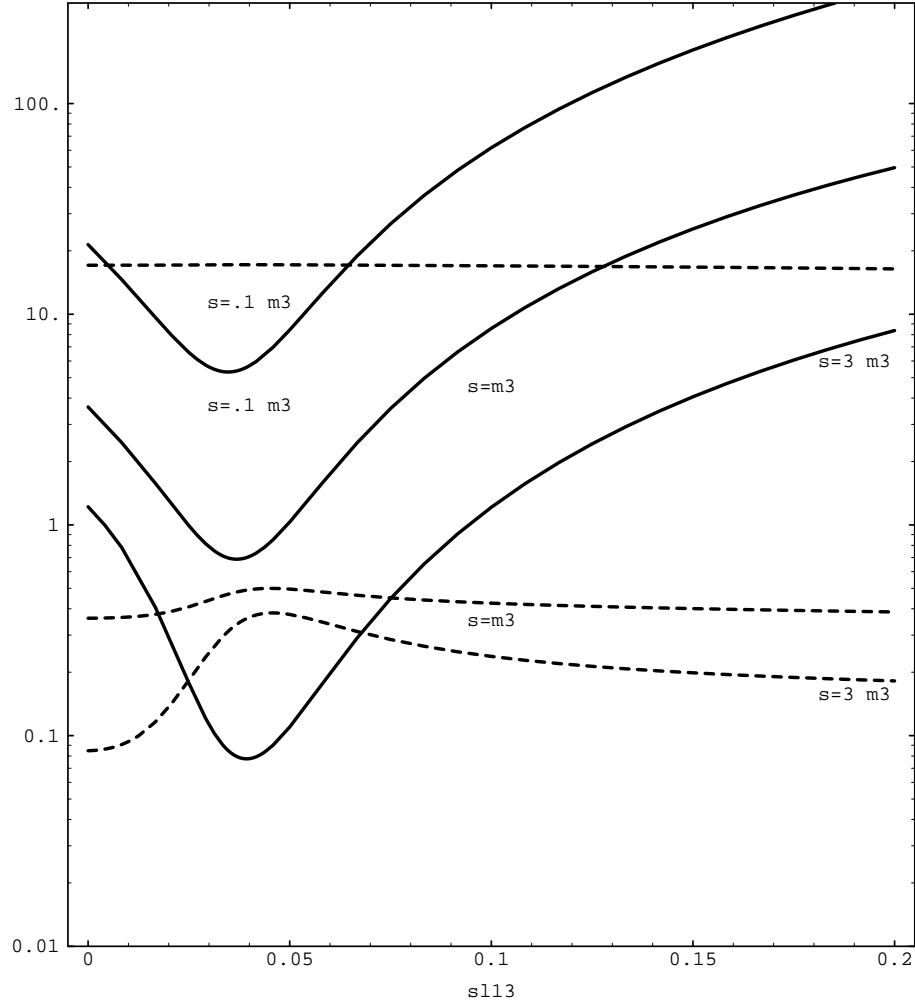


Figure 1. Variation of $10^6|\epsilon|$ (solid line) and K (dotted line) with s_{L13} for three different values of the seesaw contribution s in case of the small angle MSW solution for the solar neutrino problem. Values chosen for relevant parameters are $s_{L23} = 1/\sqrt{2}$, $s_{L12} = .04$, $m_2 = \sqrt{8 \cdot 10^{-6} \text{ eV}^2}$, $m_3 = .07 \text{ eV}$, $\lambda_2 = \pi/4$, $\delta = \pi/6$, $\lambda_1 = \pi/3$.

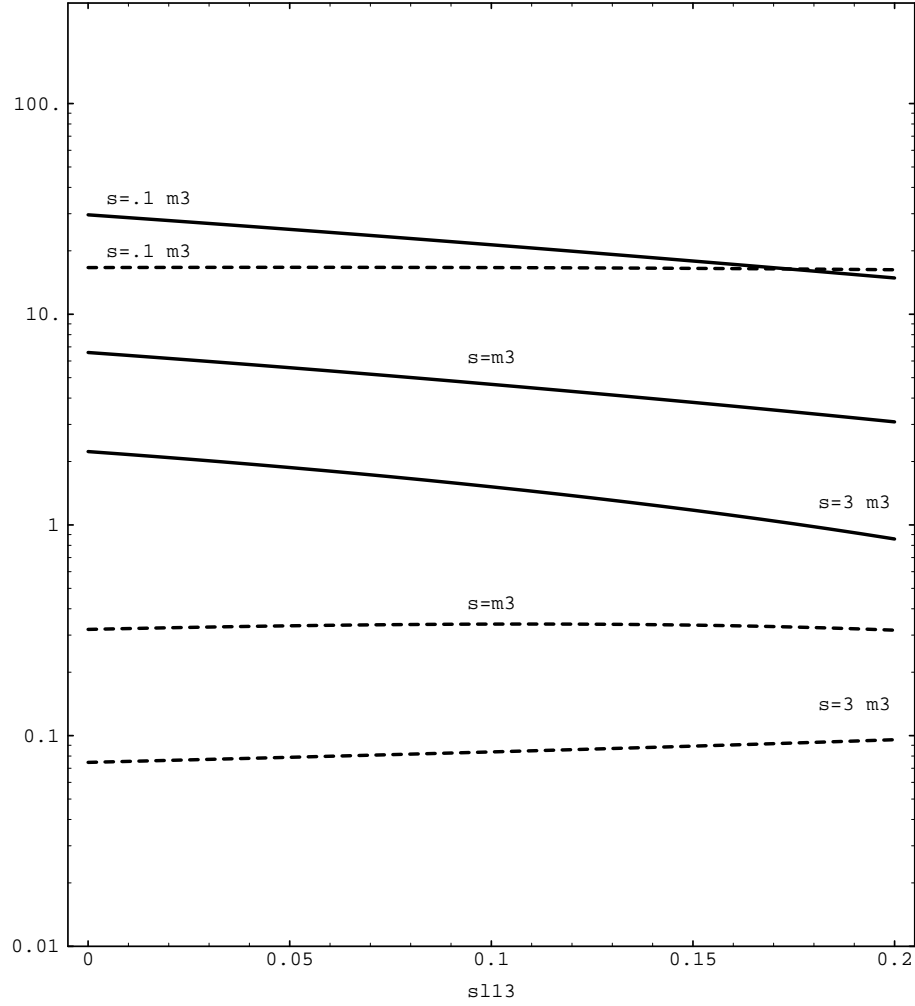


Figure 2. Variation of $10^4|\epsilon|$ (solid line) and K (dotted line) with s_{L13} for three different values of the seesaw contribution s in case of the large angle MSW solution for the solar neutrino problem. Values chosen for relevant parameters are $s_{L23} = 1/\sqrt{2}$, $s_{L12} = .5$, $m_2 = \sqrt{1.1 \cdot 10^{-5} \text{ eV}^2}$, $m_3 = .07 \text{ eV}$, $\lambda_2 = \pi/4$, $\delta = \pi/6$, $\lambda_1 = \pi/3$.

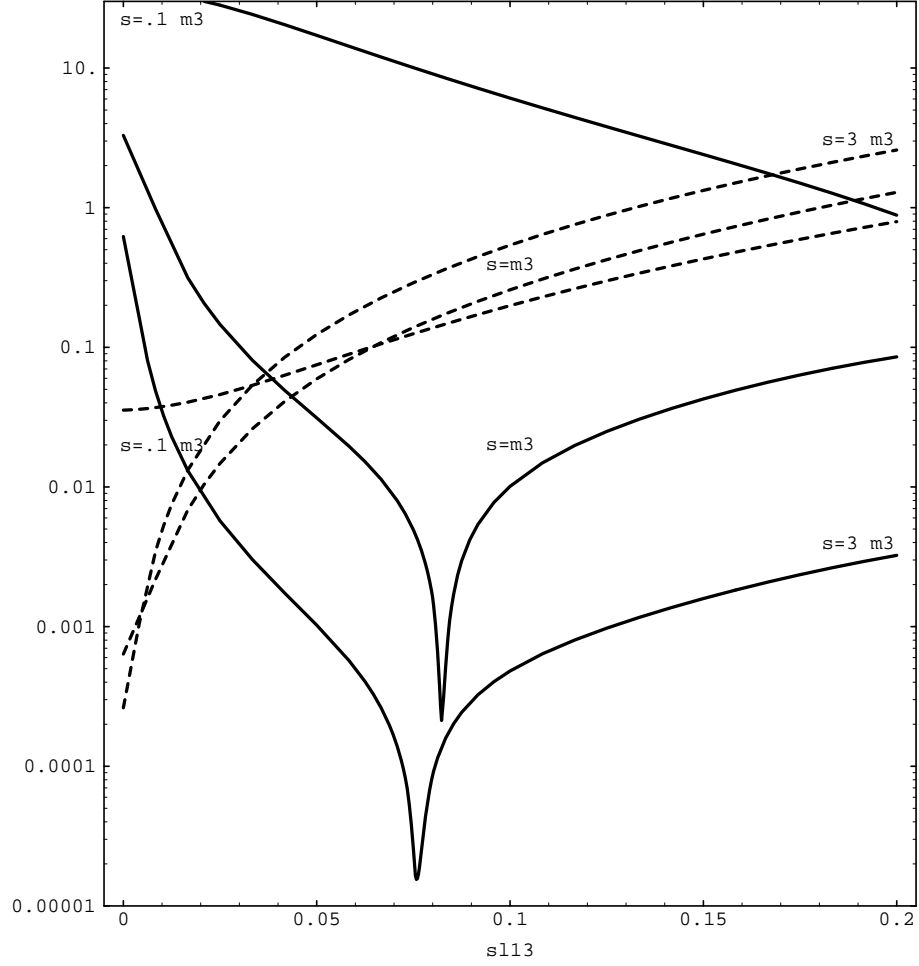


Figure 3. Variation of $10^6|\epsilon|$ (solid line) and K (dotted line) with s_{L13} for three different values of the seesaw contribution s in case of the vacuum oscillation solution for the solar neutrino problem. Values chosen for relevant parameters are $s_{L23} = s_{L12} = 1/\sqrt{2}$, $m_2 = \sqrt{8 \cdot 10^{-11}} \text{ eV}^2$, $m_3 = .07 \text{ eV}$, $\lambda_2 = \pi/4$, $\delta = \pi/6$, $\lambda_1 = \pi/3$